MENIIT

NEET | IIT-JEE | FOUNDATION

Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 28-06-2022 (MORNING SHIFT) | TIME : (9.00 AM to 12.00 PM)

Duration 3 Hours | Max. Marks : 300

QUESTIONS & SOLUTIONS

PART-PHYSICS

1. Given below are two statements : One is labelled as Assertion A and other is labelled as Reason R.

Assertion A : Product of Pressure (P) and time (t) has the same dimension as that of coefficient of viscosity.

Reason R : Coefficient of viscosity = $\frac{\text{Force}}{\text{Velocitygradient}}$

Choose the correct answer from the options given below:

A Both **A** and **R** true, and **R** is correct explanation of **A.**
B Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.
C **A** is true but **R** is false.
D **A** is false but **R** is true.

Ans. (C)

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as a = k^2rt^2, where k is a constant. The power delivered to the particle by the force acting on it is given as

(A) zero
(B) $mk^2r^2t^2$
(C) mk^2r^2t
(D) mk^2rt

Ans. (C)

3. Motion of a particle in x–y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and y = 4 sin(ωt) m. The path of the particle will be :

(A) circular
(B) helical
(C) parabolic
(D) elliptical

Ans. (A)

4. Match List-I with List-II

	List-I		List-II
(A)	Moment of inertia of solid sphere of radius R about any tangent.	I.	$\frac{5}{3}MR^2$
(B)	Moment of inertia of hollow sphere of radius (R) about any tangent.	II.	$\frac{7}{3}MR^2$
(C)	Moment of inertia of circular ring of radius (R) about its diameter.	III.	$\frac{1}{4}MR^2$
(D)	Moment of inertia of circular disc of radius (R) about any diameter.	IV.	$\frac{1}{2}MR^2$

Choose the correct answer from the options given below:

Ans. (A)

5. Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?

(A) $2r_A^2 = r_B^3$
(B) $r_A^3 = 2r_B^3$
(C) $r_A^3 = 4r_B^3$
(D) $T_A^2 - T_B^2 = \frac{\pi^2}{GM}\left(r_B^3 - 4r_A^3\right)$

Ans. **(C)**

6. A water drop of diameter 2 cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain is surface energy will be :
(A) 2.8×10^{-4} J
(B) 1.5×10^{-3} J
(C) 1.9×10^{-4} J
(D) 9.4×10^{-5} J

Ans. **(A)**

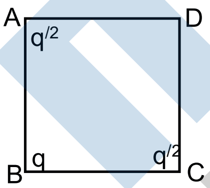
7. Given below are two statements :
Statements – I : When μ amount of an ideal gas undergoes adiabatic change from state (P_1, V_1, T_1) to state (P_2, V_2, T_2), then work done is $w = \frac{\mu R(T_2 - T_1)}{1-y}$, where $\gamma = \frac{C_p}{C_V}$ and R = universal gas constant.
Statement – II: In the above case, when work is done on the gas, the temperature of the gas would be rise.
Choose the correct answer from the options given below:
(A) Both statement-I and statement-II are true.
(B) Both statement-I and statement-II are false.
(C) Statement-I is true but statement-II is false.
(D) Statement-I is true but statement-II is true.

Ans. **(A)**

8. Given below are two statements:
Statement-I: A point charge is brought in an electric field. The value of electric field at a point near to the change may increase if the charge is positive.
Statement-II: An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.
Choose the correct answer from the options given below:
(A) Both statement-I and statement-II are true.
(B) Both statement-I and statement-II are false.
(C) Statement-I is true but statement-II is false.
(D) Statement-I is true but statement-II is true.

Ans. **(A)**

9. The three charges q/2, q and q/2 are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the corner D of the square, is :



(A) $\frac{q}{4\pi \epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$

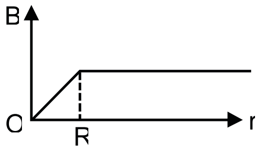
(B) $\frac{q}{4\pi \epsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}\right)$

(C) $\frac{q}{4\pi \epsilon_0 a^2}\left(1-\frac{1}{\sqrt{2}}\right)$

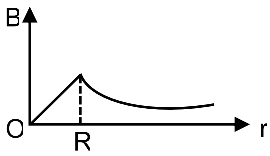
(D) $\frac{q}{4\pi \epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}-\frac{1}{2}\right)$

Ans. **(A)**

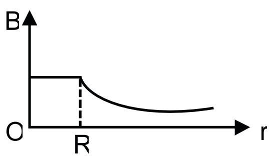
10. An infinitely long hollow conducting cylinder with radius R carries a uniform current along its surface.
Choose the correct representation of magnetic field (B) as a function of radial distance (r) from the axis of cylinder.
(A)



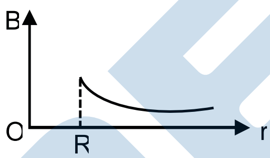
(B)



(C)



(D)



Ans. (D)

11. A radar sends an electromagnetic signal of electrics field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5×10^{-8} T which strikes a target on line of sight at a distance of 3 km in medium. After that, a part of signal (echo) reflects back towards the radar with same velocity and by same path. If the signal was transmitted at time t=0 from radar, then after how much time echo will reach to the radar?
(A) 2.0×10^{-5} s
(B) 4.0×10^{-5} s
(C) 1.0×10^{-5} s
(D) 8.0×10^{-5} s

Ans. (B)

12. The refracting angle of a prism is A and refractive index of the material of the prism is cot (A/2). Then the angle of minimum deviation will be -
(A) 180 – 2A
(B) 90 – A
(C) 180 + 2A
(D) 180 – 3A

Ans. (A)

13. The aperture of the objective is 24.4 cm. The resolving power of this telescope, if a light of wavelength 2440 Å is used to see the object will be:
(A) 8.1×10^{6}
(B) 10.0×10^{7}

(C) 8.2×10^5
(D) 1.0×10^{-8}

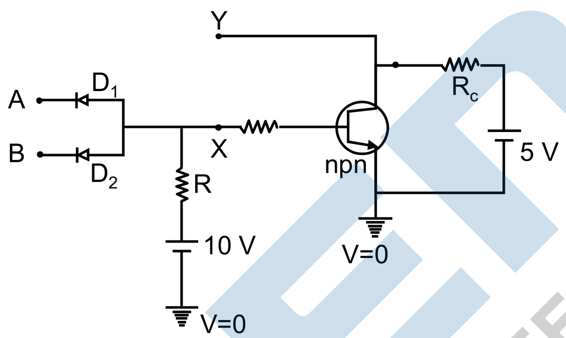
Ans. **(C)**

14. The de Broglie wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon, which of the following presents the correct relation between the de Broglie wavelengths of two ?
(A) $\lambda_p \propto \lambda^2_e$
(B) $\lambda_p \propto \lambda_e$
(C) $\lambda_p \alpha \sqrt{\lambda_e}$
(D) $\lambda_p \alpha \sqrt{\frac{1}{\lambda_e}}$

15. The Q-value of a nuclear reaction kinetic energy of the projectile particle, K_p are related as:
(A) $Q = K_p$
(B) $(K_p + Q) < 0$
(C) $Q < K_p$
(D) $(K_p + Q) > 0$

Ans. (D)

16. In the following circuit, the correct relation between output (Y) and output A and B will be:



(A) Y = AB
(B) Y = A + B
(C) $Y = \overline{AB}$
(D) $Y = \overline{A + B}$

Ans. **(C)**

17. For using a multimeter to identify diode from electrical components, choose the correct statement out of the following about the diode:
(A) It is two terminal device which conducts current in both directions.
(B) It is two terminal device which conducts current in one direction only.
(C) It does not conduct gives and initial deflection which decays to zero.
(D) It is three terminal device which conducts current in one direction only between central terminal and either of the remaining two terminals

Ans. **(B)**

18. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R
Assertion A: n-p-n transistor permits more current than p-n-p transistor.
Reason R: Electrons have grater mobility as a charge carrier.
Choose the correct answer from the options given below:
(A) Both A and R true, and R is correct explanation of A.
(B) Both A and R are true but R is NOT the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans. **(A)**

19. Match List-I with List-II

	List-I		List-II
A.	Television signal	I.	$\frac{5}{3}MR^2$
B.	Radio signal	II.	$\frac{7}{3}MR^2$
C.	High Quality Music	III.	$\frac{1}{4}MR^2$
D.	Human sppech	IV.	$\frac{1}{2}MR^2$

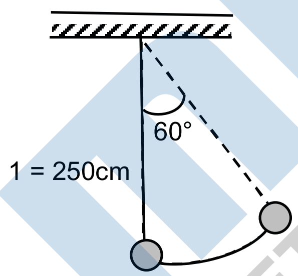
Choose the correct answer from the options given below:
(A) A-I, B-II, C-III, D-IV
(B) A-IV, B-III, C-I, D-II
(C) A-IV, B-III, C-II, D-I
(D) A-I, B-II, C-IV, D-III

Ans. (C)

20. The velocity of sound in a gas, in which two wavelengths 4.08m and 4.16m produce 40 beats in 12s, will be:
(A) 282.8 ms^{-1}
(B) 175.5 ms^{-1}
(C) 353.6 ms^{-1}
(D) 707.2 ms^{-1}

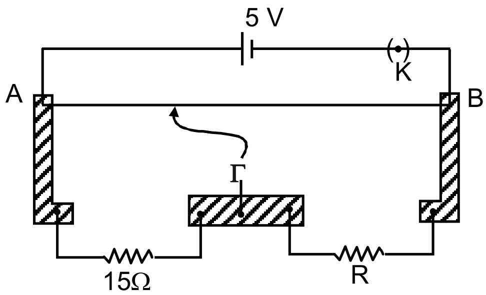
Ans. (D)

21. A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob, the maximum velocity attained by the bob will be________ ms^{-1}. (if g = 10 m/s^2)



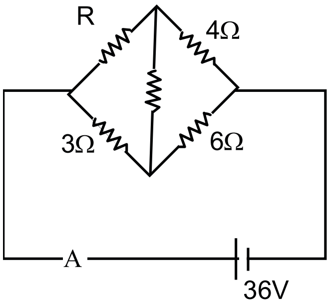
Ans. 5

22. A meter bridge setup is shown in the figure. It is used to determinate and unknown resistance R using a given resistor of 15 Ω. The galvanometer (G) shows null deflection when tapping key is at 43 cm mark from end A. If the end correction for end A is 2 cm, then the determined value of R will be __________Ω.



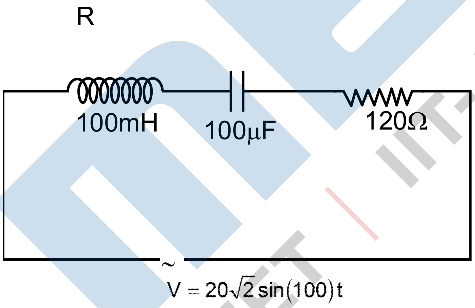
Ans. **19**

23. Current measured by the ammeter Ⓐ in the reported circuit when no current flows through 10 Ω resistance, will be _________ A.



Ans **(10)**

24. An AC source is connected to an inductance of 100 mH. a capacitance of 100 μ F and a resistance of 120 Ω as shown in figure. The time in which the resistance having a thermal capacity 2J/°C will get heated by 16°C is ___________ s.



Ans. **15**

25. The position vector of 1 kg object is $\vec{r} = \left(3\hat{j} + \hat{k}\right)$m and its velocity $\vec{v} = \left(3\hat{j} + \hat{k}\right)\text{ms}^{-1}$. The magnitude of its angular momentum is $\sqrt{x}$ Nm where is

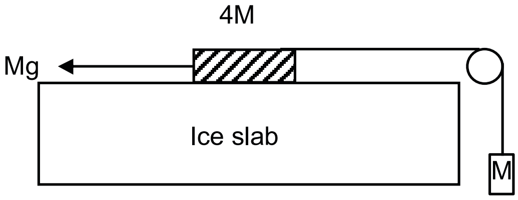
Ans. **(91)**

26. A man of 60 kg is running on the road and suddenly jumps into a stationary trolly car of mass 120 kg. Then the trolly car star5ts moving with velocity 2ms^{-1}. The velocity of the running man was _________ ms^{-1}, when he jumps into the car.

Ans. **6**

27. A hanging mass M is connected to a tour times bigger mass by using a string-pulley arrangement, as shown in the figure. The bigger mass is placed on a horizontal ice-slab and

being pulled by 2 Mg force. In this situation, tension in the string is $\frac{x}{5}$ Mg for x = ____________.
Neglect mass of the string and friction of the block (bigger mass) with ice slab.
(Given g = acceleration due to gravity)



Ans **(6)**

28. The total internal energy of two mole monoatomic ideal gas at temperature T = 300 K will be ___________ J. (Given R = 8.31 J/mol.K)

Ans. ***7479***

29. A singly ionized magnesium atom (A = 24) ion is accelerated to kinetic energy 5 keV, and is projected perpendicularly into a magnetic field B of the magnitude 0.5 T. The radius of path formed will be ____________ cm.

Ans. **10**

30. A telegram line of length 100 km has a capacity of 0.001 μF/km and it carries an alternating current 0.5 kilo cycle per second. If minimum impedance is required, then the value of the inductance that needs to be introduced in series is _____________mH. $\left(\text{if}\,\pi = \sqrt{10}\right)$

Ans. **100**

PART: CHEMISTRY

1. The incorrect statement about the imperfections in solids is:
(A) Schottky defect decreases the density of the substance.
(B) Interstitial defect increases the density of the substance.
(C) Frenkel defect does no alter the density of the substance.
(D) Vacancy defect increases the density of the substance.

Ans. (D)

Sol. vacancy defect decreases density of substance.

2. The zeta potential is related to which property of colloids?
(A) Colour
(B) Tyndall effect
(C) Charge on the surface of colloidal particles
(D) Brownian movement

Ans. (C)

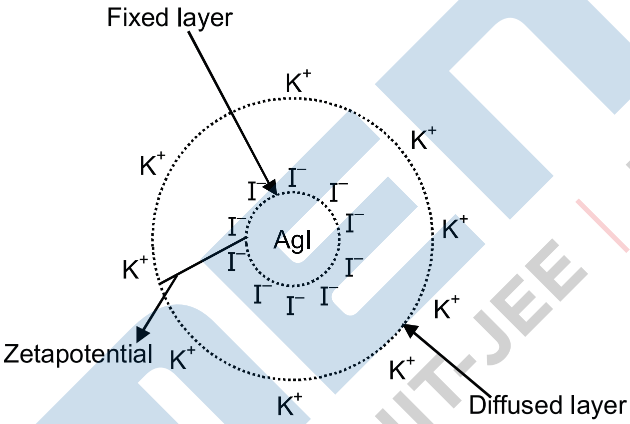
Sol. This potential difference between the fixed layer and diffused layer of opposite charges is called electrokinetic potential or zeta potential.

$z = \frac{4\pi\eta\mu}{D}$ n = viscosity coefficient

D = Dielectric constant of medium

μ = Velocity of colloidal particle when an electric field is applied.

e.g. $AgNO_3 + \underset{(excess)}{KI} \rightarrow AgI + KNO_3$



3. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above "E" in the group is
(A) $3s^2, 3p^4$
(B) $3d^{10}, 4s^2, 4p^4$
(C) $4d^{10}, 5s^2, 5p^4$
(D) $2s^2, 2p^4$

Ans. (A)

Sol.

16^{th} group		EC
2^{nd}	$_8O$	$2s^2\,2p^4$
2^{nd}	$_8S$	$3s^2\,3p^4$
4^{th}	$_{34}Se$	$4s^2\,4p^4$
5^{th}	$_{52}Te$	$5s^2\,5d^{10}\,4p^4$

4. Given are two statements one is labelled as **Assertion A** and other is labelled as **Reason R**.
Assertion A: Magnesium can reduce Al_2O_3 at a temperature below 1350°C, while above 1350°C aluminium can reduce MgO.
Reason R: The melting and boiling points of magnesium are lower than those of aluminium.

In light of the above statements, choose most appropriate answer from the options given below:
(A) Both A and R are correct, and R is correct explanation of A.
(B) Both A and R are correct, but R is NOT the correct explanation of A.
(C) A is correct R is not correct.
(D) A is not correct, R is correct

Ans. **(B)**

Sol. Below 1350°C graph of $\Delta G°$ v/s T of MgO is lower than Al_2O_2 while above 1350°C graph of MgO is above than Al_2O_2 so assertion is true.

	Al	Mg
M.P.	933 K	924 K
B.P.	2740 K	1363 K

5. Dihydrogen reacts with CuO to give
(A) CuH_2
(B) Cu
(C) Cu_2O
(D) $Cu(OH)_2$

Ans. **(B)**

Sol. $CuO + H_2 \longrightarrow Cu + H_2O$

6. Nitrogen gas is obtained by thermal decomposition of
(A) $Ba(NO_3)_2$
(B) $Ba(N_3)_2$
(C) $NaNO_2$
(D) $NaNO_3$

Ans. **(B)**

Sol. $Ba(N_3)_2 \xrightarrow{\Delta} Ba + N_2$

$2NaNO_2 \xrightarrow{\Delta} Na_2O + NO + NO_2$

$NaNO_3 \xrightarrow{\Delta} NaNO_2 + O_2$

$Ba(NO_3)_2 \xrightarrow{\Delta} BaO + NO_2D + O_2$

7. Given below are two statements:
Statement I : The pentavalent oxide of group-15 element, E_2O_3, of the element,
Statement II: The acidic character of trivalent oxide of group 15 element, E_2O_3, decreases down the group.
In light of the above statements, choose most appropriate answer from the options given below:
(A) Both statement I and Statement II are true.
(B) Both statement I and statement II are false.
(C) Statement I true, but statement II is false.
(D) Statement I is false but statement II is true.

Ans. **(D)**

Sol. S_1: In 15^{th} group oxide higher the oxidation state of element higher is acidic character
Example: P_2O_3 less acidic than P_2O_5.
15^{th} group
N_2O_3 Acidic
P_2O_3 Acidic
As_2O_3 Amphoteric
Sb_2O_3 Amphoteric
Bi_2O_3 Basic

8. Which one of the lanthanoids given below is the most stable in divalent from?
(A) Ce (Atomic Number 58)
(B) Sm (Atomic Number 62)
(C) Eu (Atomic Number 63)

(D) Yb (Atomic Number 70)

Ans. **(C)**

Sol. $_{63}Eu = [Xe]4f^7\ 6s^2$ $_{62}Sm = [Xe]4f^6\ 6s^2$ $_{65}Tb = [Xe]4f^9\ 6s^2$ $_{58}Ce = [Xe]4f^1\ 5d^1\ 6s^2$

$_{63}Eu^{2+} = [Xe]4f^7$ $_{62}Sm^{2+} = [Xe]4f^6$ $_{65}Tb^{2+} = [Xe]4f^9$ $_{58}Ce^{2+} = [Xe]4f^1\ 5d^1$

9. Given below are two statements:

Statement I: $[Ni(CN)_4]^{2-}$ is square planer and diamagnetic complex, with dsp^2 hybridization for Ni but $[Ni(CO)_4]$ is tetrahedral, paramagnetic and with sp^3-hybridization for Ni.

Statement II: $[NiCl_4]^{2-}$ and $[Ni(CO)_4]$ both have same d-electron configuration, have same geometry and are paramagnetic.

In light the above statements, choose the correct answer from the options given below:

(A) Both statement I and Statement II are true.
(B) Both statement I and statement II are false.
(C) Statement I correct, but statement II is false.
(D) Statement I is incorrect but statement II is true.

Ans. **(B)**

Sol. $[Ni(CN)_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8\ 4s^0 \Rightarrow dsp^2$ = square planer $\Rightarrow$ diamagnetic

$[Ni(CO)_4]$ Ni $\Rightarrow 3d^8\ 4s^2 \Rightarrow 3d^{10} = sp^3 \Rightarrow$ Tetrahedral $\Rightarrow$ diamagnetic

$[NiCl_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8\ 4s^0 \Rightarrow sp^3 \Rightarrow$ Tetrahedral $\Rightarrow$ paramagnetic

10. Which amongst the following is not a preticide?

(A) DDT
(B) Organophosphates
(C) Dieldrin
(D) Sodium arsenite

Ans. **(D)**

Sol. Sodium arsenite is a herbiside.

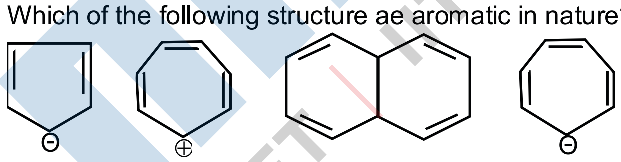
11 Which one of the following techniques is not used to spot components of a mixture separated on thin layer chromatographic plate?

(A) I_2
(B) U.V. Light
(C) Visualisation agent as a component of mobile phase
(D) Spraying of an appropriate reagent

Ans. **(C)**

Sol. It is fact.

12. Which of the following structure ae aromatic in nature?



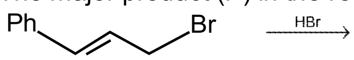
A B C D

(A) A, B, C and D
(B) Only A and B
(C) Only A and C
(D) Only B, C and D

Ans. **(B)**

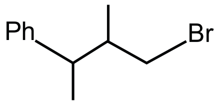
Sol. Only A and B follow Huckels rule.

13. The major product (P) in the reaction

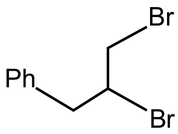


?(P) [Ph is – C_6H_5] is

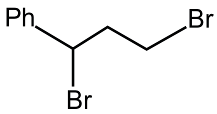
(A)



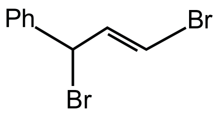
(B)



(C)

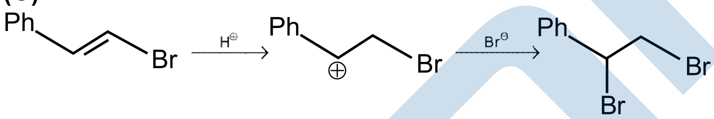


(D)



Ans. **(C)**

Sol.

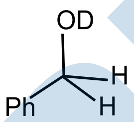


14. The correct structure of product 'A' formed in the following reaction,

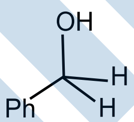
$$PhCHO + Ph \cdot CHO \xrightarrow[\text{in}D_2O]{NaOD} A + Ph-\overset{\overset{\large O}{\|}}{C}-O^- \quad (Ph \text{ is } -C_6H_5)$$

is

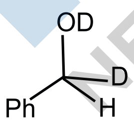
(A)



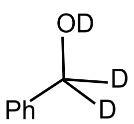
(B)



(C)

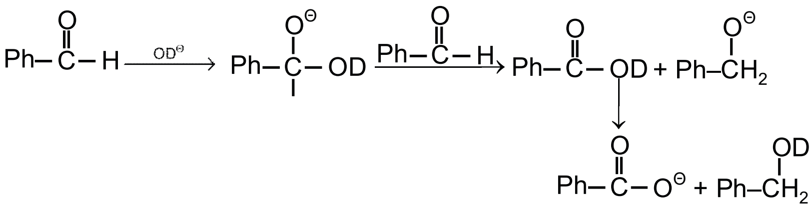


(D)



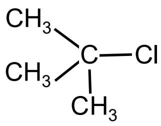
Ans. **(A)**

Sol.

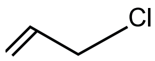


15. Which one of the following compounds is inactive towards S_N 1 reaction?

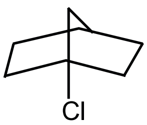
(A)



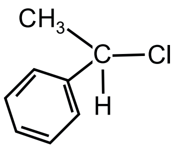
(B)



(C)



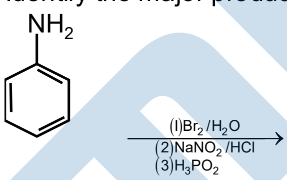
(D)



Ans. **(C)**

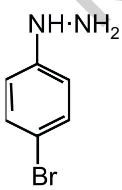
Sol. Bridge head carbocation formed in SN^1 pathway is unstable.

16. Identify the major product formed in the following sequence of reactions:

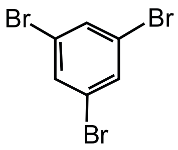


(A)

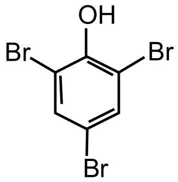
(B)



(C)

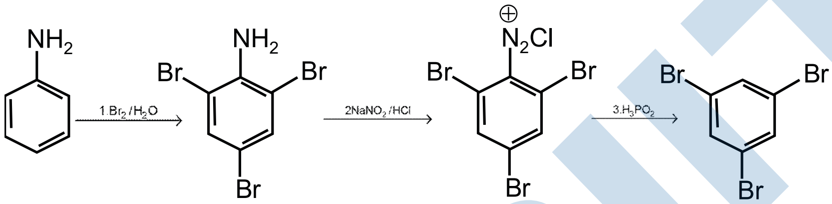


(D)



Ans. **(C)**

Sol.



17. A primary aliphatic amine on reaction with nitrous acid in cold (273 K) and there after raising temperature of reaction mixture to room temperature (298 K), gives a/an

(A) nitrile

(B) alcohol

(C) diazonium salt

(D) secondary amine

Ans. **(B)**

Sol. $R-NH_2 \longrightarrow R-N_2^+Cl^- \xrightarrow{298K} R-OH$

18. Which one of the following is NOT a copolymer?

(A) Buna-s

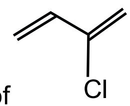
(B) Neoprene

(C) PHBV

(D) Butadiene-styrene

Ans. **(B)**

Sol. Neoprene is an addition homo polymer of



19. Stability of α - Helix structure of proteins depends upon

(A) dipolar interaction

(B) H-bonding interaction

(C) van der Waals forces

(D) π - stacking interaction

Ans. **(B)**

Sol. Stability of α-Helix structure of proteins depend upon H-bonding interactions.

20. The formula of the purple colour formed in Laissaigne's test for sulphur using sodium nitroprusside is

(A) $NaFe[Fe(CN)_6]$

(B) $Na[Cr(NH_3)_2(NCS)_4]$

(C) $Na_2[Fe(CN)_5(NO)]$

(D) $Na_4[Fe(CN)_5(NOS)]$

Ans. **(D)**

Sol. The formula of purple colour formed in Laissaine test for sulphur using Sodium nitroprusside is due to $Na_4[Fe(CN)_5(NOS)]$

21. A 2.0 g sample containing MnO_2 is treated with HCl liberating Cl_2. The Cl_2 gas is passed into a solution of KI and 60.0 mL of 0.1 M $Na_2S_2O_3$ is required to titrate the liberated iodine. The percentage of MnO_2 in the sample is ________ . (Nearest integer)

Ans. (13)

Sol. $MnO_2 + 4HCl \longrightarrow MnCl_2 + Cl_2(g) + 2H_2O$

$Cl_2 + 2KI \longrightarrow I_2 + 2KCl$

$I_2 + 2Na_2S_2O_3 \longrightarrow 2NaL + Na_2S_4O_6$

Mili eq. of MnO_2 = Mili eq. of Cl_2 = Milli eq. of I_2 = Mili eq. of Hypo.

$$2\left(\frac{w}{87}\right) = [0.1 \times 60]$$

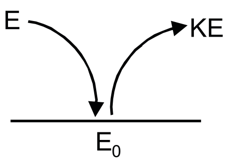
W = 261 miligram

$$\% \text{ of } MnO_2 = \frac{0.261}{2} \times 100 = 13.05\% \approx 13$$

22. If the work function of a metal is 6.63×10^{-19}J, the maximum wavelength of the photon required to remove a photoelectron form the metal is _______ nm. (Nearest integer)

Ans. (300)

Sol.



E = E + KE

for just ejection of electron

$$E = E_0 = \frac{hc}{\lambda} = 6.63 \times 10^{-34}$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} = 6.63 \times 10^{-19}$$

$\lambda = 3 \times 10^{-7}$ m

$= 300 \times 10^{-9}$ m

= 300 nm

23. The hybridization of P exhibited in PF_5 is sp^xd^y. The value of y is _______

Ans. (1)

Sol. Hybridization of PF_5 is $\Rightarrow sp^3d$.

so y = 1

24. 4.0 L of an ideal gas is allowed to expand isothermally into vacuum until the total volume is 2.0 L. The amount of heat absorbed in this expansion is ________ L atm.

Ans. (0)

Sol. Work done against vacuum = 0

As process is isothermal so $\Delta U = 0$

$\Delta U = q + w$

so q = 0

25. The vapour pressure of two voltaic liquids A and B at 25°C are 50 Torr and 100 Torr, respectively. If the liquid mixture contains 0.3 mole fractions of A, then the mole fraction of liquid B in the vapour phase is $\frac{x}{17}$. The value of x is ________ .

Ans. (14)

Sol. $P_{Total} = P^0_{A \times A} + P^0_{B \times B}$

= (50) 0.3 + (100) 0.7

= 15 + 70

$P_B = (P_{Total}) Y_B$

$\Rightarrow Y_B = \frac{78}{85} = \frac{14}{17} = 14$

26. The solubility product of a sparingly soluble salt A_2X_3 is 1.1×10^{-23}. If specific conductance of the solution is 3×10^{-5} S m^{-1}, the limiting molar conductivity of the solution is $x \times 10^{-3}$ S m^2 mol^{-1}. The value of x is _________ .

Ans. **(3)**

Sol. $K_{sp}(A_2X_2) = 1.1 \times 10^{-23} = 110 \times 10^{-25}$

$K_{sp} = (2)^2(3)^3(s)^5 = 110 \times 10^{-25}$

$4 \times 27(s)^5 = 110 \times 10^{-5}$; $S = M = 1 \times 10^{-5}$

K = 3 × 10–5 Sm^{-1}

$\lambda m = [x] \times 10^{-3}\ Sm^2mol^{-1}$

$\lambda m = \left[\frac{K \times 10^{-3}}{M}\right]$

$= \frac{3 \times 10^{-5} \times 10^{-3}}{1 \times 10^{-5}} = 3 \times 10^{-3} Sm^2mol^{-1}$

27. The quantity of electricity in Faraday needed to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} is _______

Ans. **(6)**

Sol. $Cr_2O_7{}^{2-} + 14H^+ = 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

1 mole 6mole

Charge = 6F

28. For a first reaction $A \rightarrow B$, the rate constant, $k = 5.5 \times 10^{-14}\ s^{-1}$. The time required for 67% completion of reaction $x \times 10^{-1}$ times the half life of reaction. The value of x is ________(Nearest Integer) (Given : log 3 = 0.4771)

Ans. **(16)**

Sol. $t_{67\%} = \frac{1}{k}\ln\left(\frac{100}{33}\right)$

$t_{50\%} = \frac{1}{k}\ln(2)$

$\frac{t_{67\%}}{t_{50\%}} = \frac{\ln\left(\frac{100}{33}\right)}{\ln 2}$

$\Rightarrow \frac{t_{67\%}}{t_{50\%}} = \frac{\log\left(\frac{100}{33}\right)}{\log 2}$

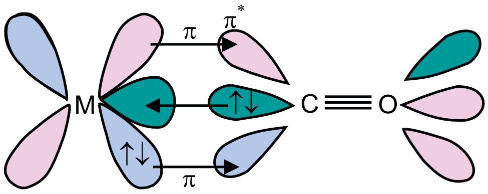
$\Rightarrow \frac{\log 3}{\log 2} = \frac{0.4771}{0.3} = 1.585$

So $x = 15.85 \approx 16$

29. Number of complex which will exhibit synergic bonding amongst, $[Cr(CO)_6]$. $[Mn(CO)_5]$ and $[Mn_2(CO)_{10}]$ is ________ .

Ans. **(3)**

Sol. The M—Cπ bond is formed by the donation of a pair electrons from a filled d orbital of metal into the vacant antibonding π^* orbital of carbon monoxide. Thus carbon monoxide acts as σ donor ($OC \rightarrow M$) and π acceptor ($OC \leftarrow M$.) with the two interactions creating a synergic effect which strengthens the bond between CO and the metal as shown in figure.



Synergic bonding

30. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide in the given compound is ________ % (nearest integer)
Relative atomic masses of Ag and Br are 108u and 80u, respectively).

Ans. (34)

Sol. Wt. of AgBr = 0.4g

$$\text{Wt. of AgBr} = \frac{0.4}{188}$$

$$\text{Wt. of Br} = \frac{0.4}{188} \times 80 \text{ g}$$

$$\% \text{ of Br} = \frac{0.4}{188} \times \frac{80}{0.5} \times 100 = 34\%$$

MATHS – PART

1. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$, Where $\alpha \in R$, then the value of 16α is equal to

(A) 1411 (B) 1320
(C) 1615 (D) 1855

Ans. (A)

Sol. $\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + \ldots + {}^{31}C_{31} \cdot {}^{31}C_{30}$

$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + \ldots + {}^{31}C_{30} \cdot {}^{31}C_0$

${}^{62}C_{30}$.

Similarly

$\sum_{R=1}^{30}\left({}^{30}C_R \cdot {}^{30}C_{R-1}\right) = {}^{60}C_{29}$

${}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!32!} - \frac{60!}{29!31!}$

$= \frac{60!}{29!31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$

$\therefore 16\alpha = 16 \times \frac{2822}{32} = 1411$

1. Let a function $f : \square \to \square$ be defined by $f(n) = \begin{cases} 2n, & n = 2,4,6,8,\ldots.. \\ n-1 & n = 3,7,11,15,\ldots.. \\ \frac{n+1}{2} & n = 1,5,9,13,\ldots.. \end{cases}$ then, f is

(A) one-one but not onto
(B) onto but not one-one
(C) neither one-one nor onto
(D) one-one and onto

Ans. (D)

Sol. $f(x) = \begin{cases} 4R, & n = 2R \\ 4R-2 & n = 4R \\ 2R-1 & n = 4R-3 \end{cases}$

$(R \in N)$

Note that for any element, it will fall into exactly one value of n.

Thus, f is one – one & onto.

3. If the system of linear equations

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda| z = 4\lambda - 4$

where $\lambda \in \square$, has no solution, then

(A) $\lambda = 7$ (B) $\lambda = -7$
(C) $\lambda = 8$ (D) $\lambda^2 = 1$

Ans. (B)

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$

$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$ ...(1)
System:
$2x + 3y - z = -2$ ...(2)
$x + y + z = 4$ ...(3)
$x - y + |\lambda| z = 4\lambda - 4$ ...(4)
Eliminating y from equal (2) & (3) we get
$X + 4z = 14$ ...(5)

$(3) + (4) \Rightarrow x + \left(\frac{|\lambda| + 1}{2}\right) z = 2\lambda$ ...(6)

Clearly for $\lambda = -7$, system is inconsistent.

4. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det(A) adj (5 adj (A^3))) is equal to ______.

(A) 512×10^6 (B) 256×10^6
(C) 1024×10^6 (D) 256×10^{11}

Ans. **(A)**

Sol. |(det (A)) adj (5 adj (A))|
$= |2\text{adj}(5\text{adj}(A^3))|$
$= 2^3 |\text{adj}(5 \text{ adj}(A^3))|$
$= 2^3 . |5\text{adj}(A^3)|^2$
$= 2^3 (5^3 . |\text{adj}(A^3)|)^2$
$= 2^3 . 5^6 . |\text{adj}A^3|^2$
$= 2^3 . 5^6 ((|A|^3)^2)^2$
$= 2^3 . 5^6 . 2^{12} = 2^{15} \times 5^6$
$= 2^9 \times 10^6$
512×10^6.

5. The total number of 5-digit numbers, formed by using the digits 1,2,3,5,6,7 without repetition, which are multiple of 6, is

(A) 36 (B) 48
(C) 60 (D) 72

Ans. **(D)**

Sol. To make a no. divisible by 3 we can use the digits 1,2,5,6,7 ro 1,2,3,5,7.
Using 1,2,5,6,7, number of even number is
$= 4 \times 3 \times 2 \times 1 \times 2 = 48$
Using 1,2,3,5,7, number of even numbers is
$= 4 \times 3 \times 2 \times 1 \times 1 = 24$
Required answer is 72.

6. Let $A_1, A_2, A_3, \ldots$ be an increasing geometric progression of positive real numbers. If $A_1 A_3 A_5 A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then the value of $A_6 + A_8 + A_{10}$ is equal to

(A) 33 (B) 37
(C) 43 (D) 47

Ans. **(C)**

Sol. $A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$

$(A_4)^4 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$

$A_2 + A_4 = \frac{7}{36}$

$A_2 = \frac{1}{36}$

$A_6 = 1$

$A_8 = 6$

$A_{10} = 36$

$A_6 + A_8 + A_{10} = 43$

7. Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 \left[-8x^2+6x-1\right]dx$ is equal to

(A) -1

(B) $-\frac{5}{4}$

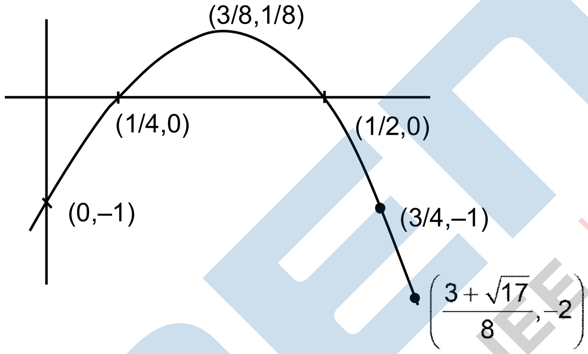
(C) $\frac{\sqrt{17}-13}{8}$

(D) $\frac{\sqrt{17}-16}{8}$

Ans. **(C)**

Sol. $\int_0^1 \left[-8x^2+6x-1\right]dx$

$= \int_0^{1/4} -1dx + \int_{1/4}^{1/2} 0dx + \int_{1/2}^{3/4} -1dx$



$\int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$

$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$

$= -\left(\frac{1}{4}-0\right) - \left(\frac{3}{4}-\frac{1}{2}\right) - 2\left(\frac{3+\sqrt{17}}{8}-\frac{3}{4}\right) - 3\left(1-\frac{3+\sqrt{17}}{8}\right)$

$= -\frac{1}{4} - \frac{1}{4} + \frac{-6-2\sqrt{17}}{8} + \frac{3}{2} - + \frac{9+3\sqrt{17}}{8}$

$\frac{\sqrt{17}-13}{8}$

8. Let f: □ → □ be defined as $f(x)=\begin{cases}[e^x], & x<0\\ ae^x+[x-1], & 0\le x<1\\ b+[\sin(\pi x)], & 1\le x<2\\ [e^{-x}]-c, & x\ge 2\end{cases}$

where a, b, c ∈ □ and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true?

(A) There exists a,b,c ∈ □ such that f is continuous of □.
(B) If f is discontinuous at exactly one point, then a + b + c = 1.
(C) If f is discontinuous at exactly one point, then a + b + c ≠ 1.
(D) f is discontinuous at atleast two points, for any values of a, b and c.

Ans. **(C)**

Sol. f(x) is discontinuous at x = 1
For continuous at x = 0; a = 1
For continuous at x = 2; b + c = 1
a + b + c =2

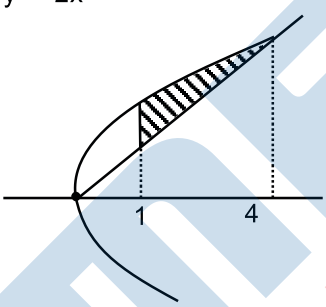
9. The area of the region $s=\{(x,y): y^2\le 8x, y\ge \sqrt{2}x, x\ge 1\}$ is

(A) $\frac{13\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{5\sqrt{2}}{6}$ (D) $\frac{19\sqrt{2}}{6}$

Ans. **(B)**

Sol. $y^2 = 8x$ ...(1)
$y=\sqrt{2}x$ ...(2)
$y^2 = 2x^2$



$\Rightarrow 8x = 2x^2$
$\Rightarrow x = 0 \ \& \ 4$

Area: $= \int_1^4 2\sqrt{2}\sqrt{x}-\sqrt{2}x\,dx$

$= 2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{3/2}\right)_1^2 - \sqrt{2}\left(\frac{x^2}{2}\right)_1^4$

$\frac{4\sqrt{2}}{3}(8-1)-\frac{\sqrt{2}}{3}(16-1)$

$\frac{28\sqrt{2}}{3}-\frac{15\sqrt{2}}{2}=\frac{11\sqrt{2}}{6}$

10. Let the solution curve y = y(x) of the differential equation,

$$\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]x\frac{dy}{dx}=x+\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]y$$

Pass through the points (1, 0) and $(2\alpha,\alpha)$, $\alpha>0$. Then α is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6}+\sqrt{e}-1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

(C) $\exp\left(\frac{\pi}{6}+\sqrt{e}+1\right)$ (D) $2\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

Ans. **(A)**

Sol. $$\left(\frac{x}{\sqrt{x^2-y^2}}+e\frac{y}{x}\right)x\frac{dx}{dx}=x+\left(\frac{x}{x^2y^2}+e^{\frac{y}{x}}\right)y$$

$$\Rightarrow e^{\frac{y}{x}}(xdy-ydx)+\frac{x}{\sqrt{x^2-y^2}}(xdy-ydx)=xdx$$

Dividing both side by x^2

$$\Rightarrow e^{\frac{y}{x}}\left(\frac{xdy-ydx}{x^2}\right)+\frac{x}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\frac{dy}{x}$$

Integrate both side

$$\int e^{\frac{y}{x}}d\left(\frac{y}{x}\right)+\int\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\int\frac{dy}{x}$$

$$\Rightarrow e^{\frac{y}{x}}+\sin^{-1}\left(\frac{y}{x}\right)^2=1nx+c$$

$1+0=0+c\Rightarrow c=1$

It passes through $(2\alpha,\alpha)$

$$e^{\frac{1}{2}}+\sin^{-1}\frac{1}{2}=1n2\alpha+1$$

$$\Rightarrow 1n\ 2\alpha=\sqrt{e}+\frac{\pi}{6}-1$$

$$\Rightarrow 2\alpha=e^{\left(\sqrt{e}\frac{\pi}{6}-1\right)}$$

$$\Rightarrow \alpha=\frac{1}{2}e^{\left(\frac{\pi}{6}+\sqrt{e}-1\right)}$$

11. Let y = y(x) be the solution of the differential equation $x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0, x>1$, with y (2) = –2. Then y (3) is equal to

(A) –18 (B) –12

(C) –6 (D) –3

Ans. **(A)**

Sol. $$x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0$$

$$x(1-x^2)\frac{dy}{dx}+(3x^2-1)y=4x^3$$

$$\frac{dy}{dx}+\frac{\left(3x^2-1\right)}{\left(x-x^3\right)}y=\frac{4x^3}{\left(x-x^3\right)}$$

$$\frac{dy}{dx}+Py=Q$$

$$IF=e^{\int Pdx}=e^{\int\frac{3x^2-1}{x-x^3}dx}$$

$$x-x^3=t\Rightarrow IF=e^{\int\frac{-dt}{t}}$$

$$=e^{-\ell nt}=\frac{1}{t}$$

$$\therefore\ IF=\frac{1}{x-x^3}$$

$$Y\times IF=\int Q\times IFdx$$

$$y\left(\frac{1}{x-z^3}\right)=\int\frac{4x^3}{x-x^3}\times\frac{1}{\left(x-x^3\right)}dx$$

$$\int\frac{4x^3}{\left(x-x^3\right)^2}dx$$

$$\int\frac{4x}{\left(1-x^2\right)^2}dx \qquad 1-x^2=K$$

$$2\int\frac{-dk}{K^2} \qquad -2xdx=dK$$

$$=-2\left(-\frac{1}{K}\right)+c$$

$$\frac{x}{x-x^3}=\frac{2}{K}+c$$

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+c$$

At x = 2, y = –2

$$\frac{-2}{2-8}=\frac{2}{1-4}+c$$

$$\frac{1}{3}=\frac{-2}{3}+c$$

$\therefore$ C = 1

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+1$$

Put x = 3

$$\frac{y}{3-27}=\frac{2}{1-9}+1$$

$$\frac{y}{-24}=-\frac{1}{4}+1$$

$$\frac{y}{-24}=\frac{3}{4}$$

$$y=\frac{3}{4}(-24)=-18$$

12. The number of real solutions of $x^7+5x^3+3x+1=0$ is equal to __________.

(A) 0
(B) 1

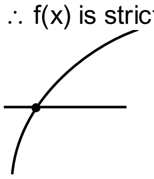
(C) 3 (D) 5

Ans. **(B)**

Sol. $f(x) = x^7 + 5x^3 + 3x + 1$

$f'(x) = 7x^6 + 15x^2 + 3 > 0$

$\therefore$ f(x) is strictly increasing function



$X \to -\infty,\ y \to -\infty$

$X \to \infty,\ y \to \infty$

$\therefore$ no. of real solution = 1

13. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If y = 2x + c is a tangent to the hyperbola H, then the value of c^2 is equal to

(A) 18 (B) 20

(C) 24 (D) 32

Ans. **(B)**

Sol. $y = mx \pm \sqrt{a^2m^2 - b^2}$

$m = 2, c^2 = a^2m^2 - b^2$

$C^2 = 4a^2 - b^2$

$e^2 = 1 + \frac{b^2}{a^2}$

$\frac{5}{2} = 1 + \frac{b^2}{a^2}$

$\frac{3}{2} = \frac{b^2}{a^2} \Rightarrow b^2 = \frac{3a^2}{2}$

$\frac{2b^2}{a} = 6\sqrt{2}$

$\frac{2}{a} = 1 + \frac{3a^2}{2} = 6\sqrt{2}$

$3a = 6\sqrt{2}$

$a = 2\sqrt{2}$

$b^2 = \frac{3}{2} \times 8 = 12$

$b = 2\sqrt{3}$

$\therefore c^2 = 4 \times 8 - 12$

$c^2 = 20$

14. If the tangents drawn at the point O(0, 0) and $P(1+\sqrt{5}, 2)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Ans. **(C)**

Sol. Tangent at O

– (x + 0) –2 (y + 0) = 0

⇒ x + 2y = 0

Tangent at P

$x\left(1+\sqrt{5}\right)+y.2-\left(x+1+\sqrt{5}\right)-2\left(y+2=0\right)$

Put x = –2y

$-2y\left(1+\sqrt{5}\right)+2y+2y-1-\sqrt{5}-2y-4=0$

$2\sqrt{5}y=5+\sqrt{5} \Rightarrow y=\left(\frac{\sqrt{5}+1}{2}\right)$

$Q\left(\sqrt{5}+1,\frac{\sqrt{5}+1}{2}\right)$

Length of tangent $OQ=\frac{5+\sqrt{5}}{2}$

$\text{Area}=\frac{RL^3}{R^2+L^2}$

$R=\sqrt{5}$

$$\frac{\sqrt{5}\times\left(\frac{5+\sqrt{5}}{2}\right)^3}{5+\left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$\frac{\sqrt{5}}{2}\times\frac{4\times\left(125+75+75\sqrt{5}+5\sqrt{5}\right)}{\left(20+25+10\sqrt{5}+5\right)}$$

$$=\frac{5+3\sqrt{5}}{2}$$

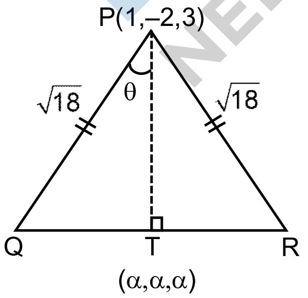
15. If two distinct point Q, R lie on the line on intersection of the planes –x + 2y – z = 0 and 3x – 5y + 2z = 0 and $PQ=PR=\sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Ans. (B)

Sol.



–x + 2y – z = 0

3x – 5y + 2z = 0

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1)$

$\vec{n} = -\hat{i} - \hat{j}(1) + \hat{k}(-1)$

Equation of LOI is $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

DR: of PT $\rightarrow \alpha - 1, \alpha + 2, \alpha - 3$

DR: of QR $\rightarrow$ 1, 1, 1

$\Rightarrow (\alpha - 1) \times 1 + (\alpha + 2) \times 1 + (\alpha - 3) \times 1 = 0$

$3\alpha = 2$

$\alpha = \frac{2}{3}$

$PT^2 = \frac{1}{9} + \frac{64}{9} + \frac{49}{9}$

$PT^2 = \frac{114}{9}$

$PT = \frac{114}{3}$

$\cos\theta = \frac{\sqrt{114}}{3} \times \frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19 \times 3}}{3 \times 3} = \frac{\sqrt{19}}{3\sqrt{3}}$

$\cos\theta = \frac{\sqrt{114}}{3} \times \frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19 \times 3}}{3 \times 3} = \frac{\sqrt{19}}{3\sqrt{3}}$

$\cos 2\theta = \frac{2 \times 19}{27} - 1 = \frac{11}{27}$

$\frac{7}{2}$

$\text{Area} = \frac{1}{2} \times \sqrt{18}\sqrt{18} \times \frac{4}{27}\sqrt{38}$

$= \frac{18}{2} \times \frac{4}{27}\sqrt{38} = \frac{36}{27}\sqrt{38} = \frac{4}{3}\sqrt{38}$

16 The acute angle between the planes P_1 and P_2 are the planes passing through the intersection of the planes 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the points (2, 1, 3) and (0, 1, 2,), respectively, is

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{12}$

Ans. **(A)**

Sol. Equation of plane passing through the intersection of planes 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 is

5x + 8y + 3z – 29 + λ (8x – 7y + z – 20) = 0 and if it is passing through (2,1,3) then $\lambda \frac{7}{2}$

P_1: Equation of plane through intersection of 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the point (2,1,3) is

5x + 8y + 3z – 29 + $\frac{7}{2}$(8x – 7y + z – 20) = 0

$\Rightarrow 2x - y + z = 6$

Similarly P_2: Equation of plane through intersection of

$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$

and the point (0,1,2) is

$\Rightarrow x + y + 2z = 5$

Angle between planes = $\theta = \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right) = \frac{\pi}{3}$

17 Let the plane $P : \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 6$ and $\vec{r} \cdot \left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90 (B) 93

(C) 95 (D) 97

Ans. **(B)**

Sol. Equation of plane passing through line of intersection of planes $P_1 : \vec{r} \cdot \left(\left(\hat{i} + 3\hat{j} - \hat{k}\right)\right) = 6$ and

$P_2 : \vec{r} \cdot \left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$ is

$P_1 + \lambda P_2 = 0$

$\left(\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) - 6\right) + \lambda\left(\vec{r} \cdot \left(-6\hat{i} + 5\hat{j} - \hat{k}\right) - 7\right) = 0$

and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda\left(-12 + 15 - \frac{1}{2} - 7\right) = 0$

$\Rightarrow \lambda = 1$

Equation of plane is $\vec{r} \cdot \left(-5\hat{i} + 8\hat{j} - 2\hat{k}\right) = 13$

$|\vec{a}|^2 = 25 + 64 + 4 = 93$; $d = 13$

Value of $\frac{|13\vec{a}|^2}{d^2} = 93$

18. The probability, that in randomly selected 3-digit number at least two digits are odd, is

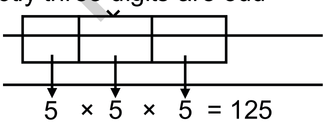
(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Ans. **(A)**

Sol. Atleast two digits are odd = exactly two digits are odd + exactly there 3 digits are odd

For exactly three digits are odd



For exactly two digits odd:

If 0 is used then : 2 × 5 × 5 = 50

If 0 is not used then : ${}^3C_1 \times 4 \times 5 \times 5 = 300$

Required Probability = $\frac{475}{900} = \frac{19}{36}$

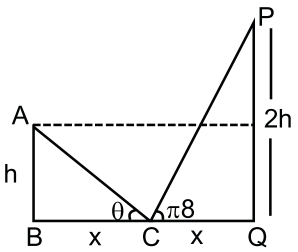
19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation form C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

Ans. **(C)**

Sol.



Let BC = CQ = x & AB = h and PQ = 2h

$\tan\theta = \frac{h}{x}, \tan\frac{\pi}{8} = \frac{2h}{x}$

$\frac{\tan\theta}{\tan\left(\frac{\pi}{8}\right)} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}, \tan\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(\sqrt{2}-1\right)$

$\tan^2\theta = \frac{1}{4}\left(3-2\sqrt{2}\right)$

20. Let p, q, r be three logical statements. Consider the compound statements

S_1: $((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

S_1: $p \rightarrow (q \vee r)$

Then, which of the following is NOT ture?

(A) If S_2 is True, then S_1 is True

(B) If S_2 is False, then S_1 is False

(C) If S_2 is False, then S_1 is True

(D) If S_1 is False, then S_2 is False

Ans. **(C)**

Sol. $S_1 : (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv \sim p \vee (q \vee r)$

S_2: $p \rightarrow (q \vee r)$

$\equiv \sim p \vee (q \vee r) \rightarrow$ By conditional law

$S_1 \equiv S_2$

21. Let R_1 and R_2 be relations on the set {1, 2,….,50} such that

$R_1 = \{(p, p^n)$: p is prime and $n \geq 0$ is an integer} and $R_2 = \{p, p^n\}$: p is a prime and n = 0 or 1}. Then, the number of elements in $R_1 - R_2$ is _____.

Ans. **(8)**

Sol. Here, $p, p^n \in \{1,2,...50\}$

Now p can take values

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43 and 47.

$\therefore$ we can calculate on. of elements in R, as

$(2, 2^{\circ}), (2,2^{1})..(2,2^{5})$

$(3,3^{\circ}),\ldots(3,3^{3})$

$(5,5^{\circ}),\ldots(5,5^{2})$

$(7,7^{\circ}),\ldots(7,7^{2})$

$(11,11^{\circ}),\ldots(11,11^{1})$

And rest for all other two elements each

$\therefore n(R_1) = 6 + 4 + 3 + 3 + (2 \times 10) = 36$

Similarly for R_2

$(2,2^{\circ}),(2,2^{1})$

$(47, 47^{\circ}),(47,47^{1})$

$\therefore n(R_2) = 2\times14 = 28$

$\therefore n(R_1) - n(R_2) = 36 - 28 = 8$

22. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$ is ________ .

Ans. (2)

Sol. $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$

$$f(x) = e^{2x}\left(e^{2x} + \frac{1}{e^{2x}} + 4\left(e^{x} + \frac{1}{e^{2x}} + 4\right) - 58\right)$$

$$e^{x} + \frac{1}{e^{x}}$$

Let $h(t) = t^2 + 4t - 58 = 0$

$$t = \frac{-4 \pm \sqrt{16 + 4.58}}{2}$$

$$\frac{-4 \pm 2\sqrt{62}}{2}$$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \geq 2$

$$e^{x} + \frac{1}{e^{x}} = -2 + 2\sqrt{62}$$

$$e^{x} - \left(-2 + 2\sqrt{62}\right)e^{x} + 1 = 0$$

$$e^{2x} - \left(-2 + 2\sqrt{62}\right)e^{x} + 1 = 0$$

$$\left(-2 + 2\sqrt{62}\right) - 4$$

$$4 + 4.62 - 8\sqrt{62} - 4$$

$$248 - 8\sqrt{62} > 0$$

$$\frac{-b}{2a} > 0$$

both roots are positive

2 real roots

23. The man and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _______.

Ans. (17)

Sol. We have

Variance = $\dfrac{\sum_{r=1}^{15} X_r^2}{15} - \left(\dfrac{\sum_{r=1}^{15} X}{15}\right)^2$

Now, the new $\sum X_r^2 = \log 5 - 5^2 + 20^2 = 1470$

And, new $\sum x_r = (15\times 8) - 5 + (20) = 135$

$\therefore$ Variance $= \dfrac{1470}{15} - \left(\dfrac{135}{15}\right)^2 = 98 - 81 = 17$

24. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$, and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, are coplanar vectors and $\vec{a}\cdot\vec{c} = 5, \vec{b} \perp \vec{c}$, then 122 $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, ($c_1 + c_2 + c_3$) is equal to _______.

Ans. (150)

Sol. $\bar{a}\cdot\bar{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5$

$\bar{b}\cdot\bar{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0$

And $\left[\bar{a}\bar{b}\bar{c}\right] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0$ …(3)

By solving (1), (2), (3) we get

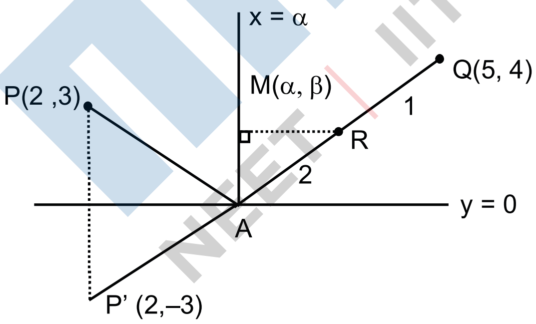
$c_1 = \dfrac{10}{122}, c_2 = \dfrac{-85}{122}, c_3 = \dfrac{225}{122}$

$\therefore 122(c_1 + c_2 + c_3) = 150$

25 A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5,4). Let R be the point that divides the line segment AQ internally into the ratio 2: 1, Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to_______.

Ans. (31)

Sol.



By observation we see that A $(\alpha, 0)$.

And β = y -coordinate of R

$\dfrac{2\times 4 + 1\times 0}{2+1} = \dfrac{8}{3}$ …(1)

Now P' is image of P in y = 0 which will be P' (2, –3)

$\therefore$ Equation of P' Q is $(y + 3) = \dfrac{4+3}{5-2}(x-2)$

i.e. $3y + 9 = 7x - 14$

$A \equiv \left(\frac{23}{7}, 0\right)$ by solving with y = 0

$\therefore \alpha = \frac{23}{7}$ ...(2)

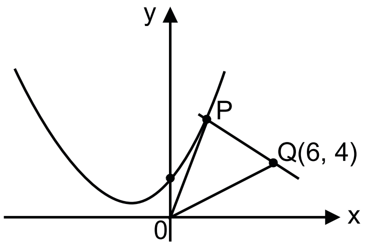
By (1), (2)

$7\alpha + 3\beta = 23 + 8 = 31$

26. Let ℓ be a line which is normal to the curve $y = 2x^2 + x + 2$ at the point P on the curve. If the point Q(6, 4) lies on the line ℓ and O is origin, then the area of the triangle OPQ is equal to ______.

Ans. **(13)**

Sol. $y = 2x^2 + x + 2$



$\frac{dy}{dx} = 4x + 1$

Let P be (h, k), then normal at P is

$y - k = -\frac{1}{4h+1}(x - h)$

This passes through Q (6, 4)

$\therefore 4 - k = -\frac{1}{4h+1}(6 - h)$

$\Rightarrow (4h + 1)(4 - k) + 6 - h = 0$

Also $k = 2h^2 + h + 2$

$\therefore (4h + 1)(4 - 2h^2 - h - 2) + 6 + h = 0$

$\Rightarrow 4h^3 - 3h^2 + 3h - 8 = 0$

$\Rightarrow h = 1, k = 5$

Now area of ΔOPQ will be $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 5 \\ 1 & 6 & 4 \end{vmatrix} = 13$

27. Let $A = \{1, a_1, a_2 \ldots\ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots\ldots + a_{18}$ is equal to ______.

Ans. **(702)**

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{18}, 77.$

Hence $a_1 + a_2 + a_3 + \ldots + a_{18} = 5 + 9 + 13 + \ldots$ 18 terms = 702

28. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 \cdot \ell$ is an odd integer, is

Ans. **(2)**

Sol. $\left(2x^3 + \frac{3}{x^k}\right)^{12}$

$t_{r+1} = {}^{12}C_r \left(2x^3\right)^r \left(\frac{3}{x^k}\right)^{12-r}$

$X^{3r-(12-r)k} \rightarrow$ constant

$\Rightarrow$ possible values of r are 3, 6, 8, 9, 10 and corresponding values of k are 1, 3, 6, 9, 15

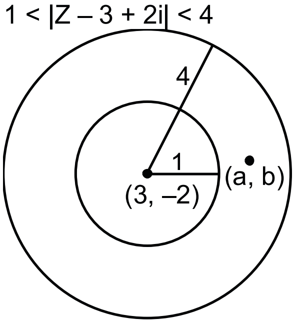
Now $^{12}C_r$ = 220, 924, 495, 220, 66

$\therefore$ possible values of k for which we will get 2^8 are 3, 6

29. The number of elements in the set {z = a + ib ∈ □ : a b ∈ □ and 1 < | z – 3 + 2i| < 4} is ________.

Ans. **(40)**

Sol. $1 < |Z - 3 + 2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$

(0, ±2), (±2, 0), (±1, ±2), (±2, ±1)

(±2, ±3), (3±, ±2), (±1, ±1), (2±, ±2)

(±3, 0), (0, ±3), (±3 ± 1), (±1, ±3)

Total 40 points

30. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle C: $(x-h)^2 + (y+k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ______.

Ans. **(816)**

Sol. Normal are

$y + 2x = \sqrt{11} + 7\sqrt{7}$,

$2y + x = 2\sqrt{11} + 6\sqrt{7}$

Center of the circle is point of intersection of normal i.e.

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$